2 copies

Ed,

This is a revised version of the report I sent you last week - temp. variations are now included.

Trip

PS This is my final, NARAM-15, version.
PPS Note my new address.

Canadian Powder

THE PRESSURE AND TEMPERATURE DEPENDENT PROPERTIES OF BLACK POWDER PROPELLANTS

CIL powder 11-28-80 edb

by the Barber Team
NAR Team #151D

18 June, 1973

Ens. A. H. Barber III
USS Alacrity (AG 520)
FPO NY
09501

Powder Comp.
74% KNO_3
10.4% S
15.6% Charcoal

Send Copy of this to Gunther Loh & to Dr. Blount

INTRODUCTION

Black powder is not a composition that has been in wide professional use as a rocket propellant during the Twentieth Century. Consequently, relatively few of the critical parameters concerning its performance as one - combustion temperature, burning rate exponent, etc. - have previously been established by modern experimental techniques. This project has attempted, with some degree of success, to define variations of the performance of black powder with the temperature and chamber pressure at which it is burned, specifically the variations in burning rate, specific impulse, and thrust. This information is of value in the design of model rocket engines, and in projects where the exact thrust of the engines must be known for a given operating temperature.

The special uncored, nozzle-less grains of Estes Industries black powder used in this project were hand-made and donated by Ed Brown of Estes, to whom we wish to express our most sincere thanks. We also wish to thank Dane Boles of Estes, who provided the project with a huge number of C6-0 engines from the same batch, at a great discount.

APPARATUS AND PROCEDURE

Two sets of measurements were made in this project: thrust, burning rate, and chamber pressure for various ratios of burning to throat area (at constant grain temperature); and thrust and burning rate for various grain temperatures (at constant area ratio).

For the first set of measurements, purely end-burning propellant charges were used. These had a fixed, planar burning surface with the cross-sectional area of the 0.5" ID tube into which they were loaded, so the only way to obtain the desired variations of pressure was to use several nozzles with different throat areas. The primary piece of apparatus used in these measurements (see Fig. 1) was a gas-tight metal casing into which could be placed one of the 2.75"x0.69" cylindrical propellant charges and one 0.75" diameter graphite nozzle section. Two tapped holes were put in the casing, one to lead to a transducer for measurement of the chamber pressure, and one for the ignition system. The whole device was mounted on a steel cantilever beam, to which strain gauges were attached to permit thrust measurements. This beam and the bottom section of the three-piece casing were used alone for the second set of measurements.

A total of six 0.6" long graphite nozzle sections were made, with their original throat diameters being those indicated by asterisks in Table 1. A very small diameter, home-made boring tool was used in conjunction with the compound feed attachment on a lathe to give each nozzle a 15° diverging section half-angle and a 45° converging section half-angle. A 60° taper on the exit end mated with a similar taper on the casing end piece when it was screwed on, to form a pressure seal.

Two of the three pieces of the casing were made of aluminum, in order to minimize the mass at the end of the cantilever and maximize its natural frequency. The base piece was made of steel, so that it could be easily silver-soldered to the steel cantilever.

The 1"x7" cantilever was made of 1/8" steel, and was rigidly clamped approximately 1" from one end. Four SR-4 strain gauges (gauge factor 2.05, resistance 120 ohms) were attached to the beam, centered 4.75" from the center of the motor casing, with two in tension and two in compression in a Wheatstone bridge arrangement. The natural frequency of the beam was 55 cycles per second with the entire casing attached, and 65 cps without the top two sections. With the pressure transducer cable attached, the frequency dropped to 28.

The ignition system for the pressure-variation tests was built around a model airplane Glo-Plug, a device which has the advantage of being built into a pressure-proof threaded bolt. This was screwed into a hole in the side of the casing, right above the end of the propellant charge. A small quantity of Ignition Mixture A (see below) was tamped into the coil on the inside of the Glo-Plug, where it was easily ignited by the wire. The sparks from this combustion quickly ignited the faster- and hotter-burning Ignition Mixture B.

Ignition Mixture A

75 % KNO_3
15 % S
10 % C

Ignition Mixture B

30 % KNO_3
9 % C
25 % $KClO_4$
9 % S
20 % $KClO_3$
7 % Mg

The device used to measure the chamber pressure was connected to the casing by an Armor-Flex 18" flexible cable, leading off from a point halfway between the end of the grain and the entrance of the

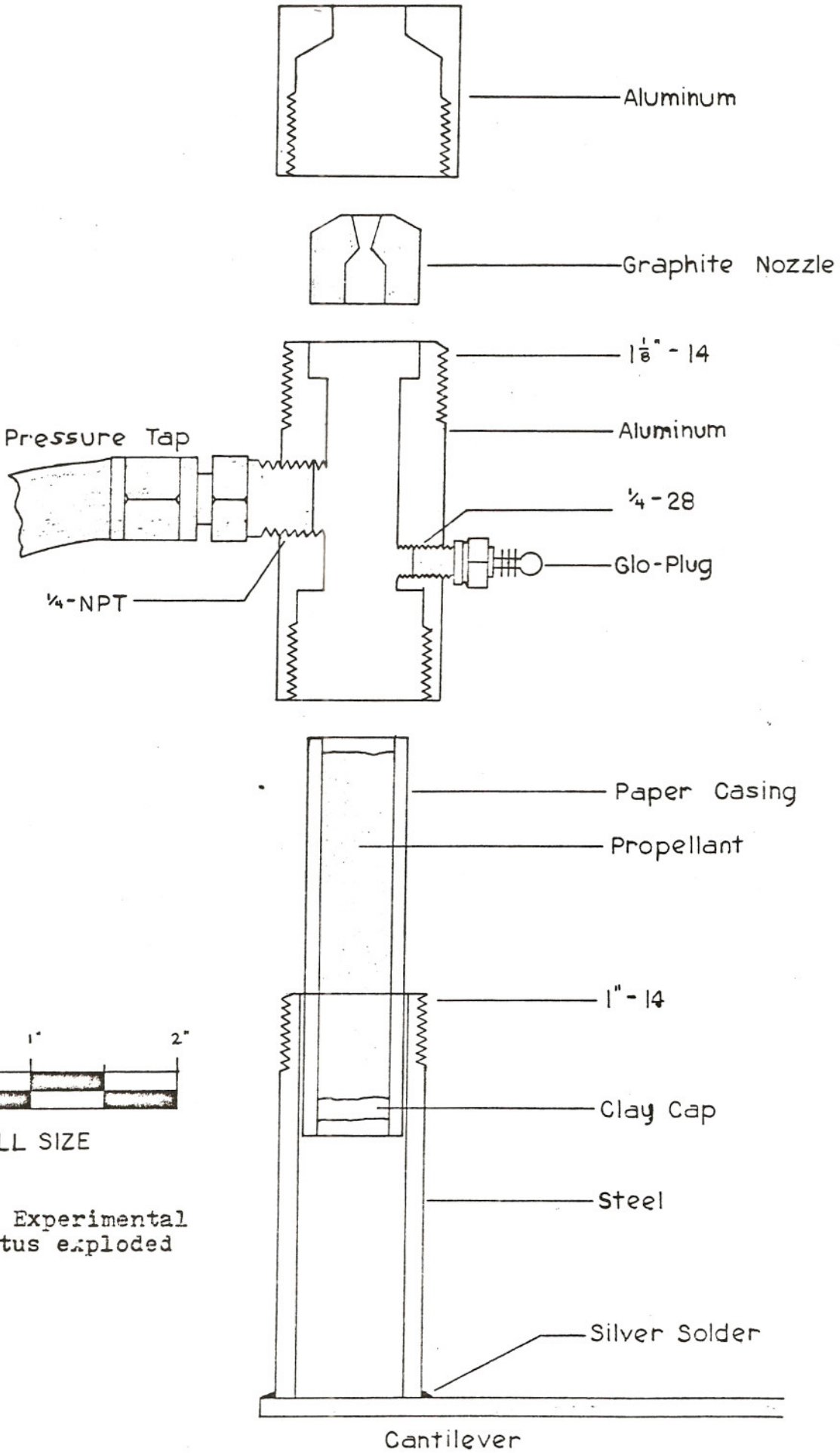
nozzle. A CEC 84145-0-800 strain-gauge type pressure transducer was used; its capacity was 800 psia, about twice that encountered at any point in this project. Because of the length of the cable connecting it to the casing (necessary to protect it from heat damage), its response time was probably about .002 sec.. Calibration was achieved by connecting the transducer to a bottle of high-pressure nitrogen with a 0-100 psig or a 0-1000 psig (depending on the required sensitivity) Bourdon tube (dial) pressure gauge placed in the line to measure pressure. Taking into account both calibration and recorder errors, the pressure measurements made in this project were probably accurate to about 5%.

A Sanborn Twin Viso Model 60, 2-channel chart recorder was used to record both thrust and pressure simultaneously, with separate Model 64-500 Strain Gauge Amplifiers providing the excitation voltage and output amplification for the two strain gauge systems. The response time of the recorder system is .007 sec., making the total response time of the pressure measurement system .009 sec., and of the thrust measurement system (because of the viscous damping system used - see Fig. 2) about .010 sec.

Calibration of the thrust-measuring system was obtained by piling pre-measured weights on the casing (centered on it) before firing. The accuracy here was about 2%, all errors being in the width of the recorder output line.

For the measurements of variations with grain temperature, 52 Estes C6-0 engines, all from the same batch, were fired after being stored at a controlled temperature for an hour. Peak thrust, sustainer thrust, and burn time were measured. Because the engines were identical, it could be assumed that the ratio of burning area to throat

area was constant. The controlled-temperature environments were made with various arrangements of dry ice, alcohol, and water for the low temperatures (41°F and below), and with a lab oven for the high temperatures (122°F and 158°F). The engines were stored in these environments with an ignitor in place, and were fired within 10 seconds of being removed, so it is believed that the initial grain temperature was no more than 5° different from the estimated (environmental) value. Conventional model rocket ignition techniques were used for these motors, since the pressure fittings were not used.



FULL SIZE

FIGURE 1. Experimental apparatus exploded view.

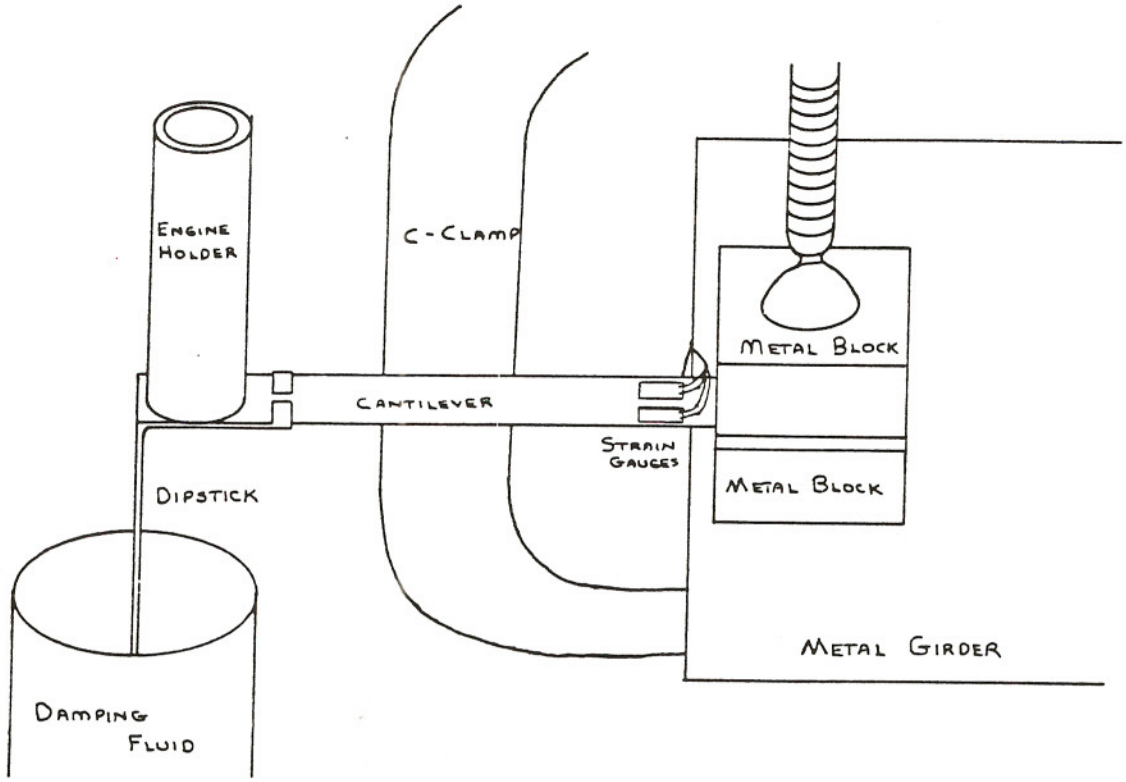


FIGURE 2. Apparatus for thrust-only measurements.

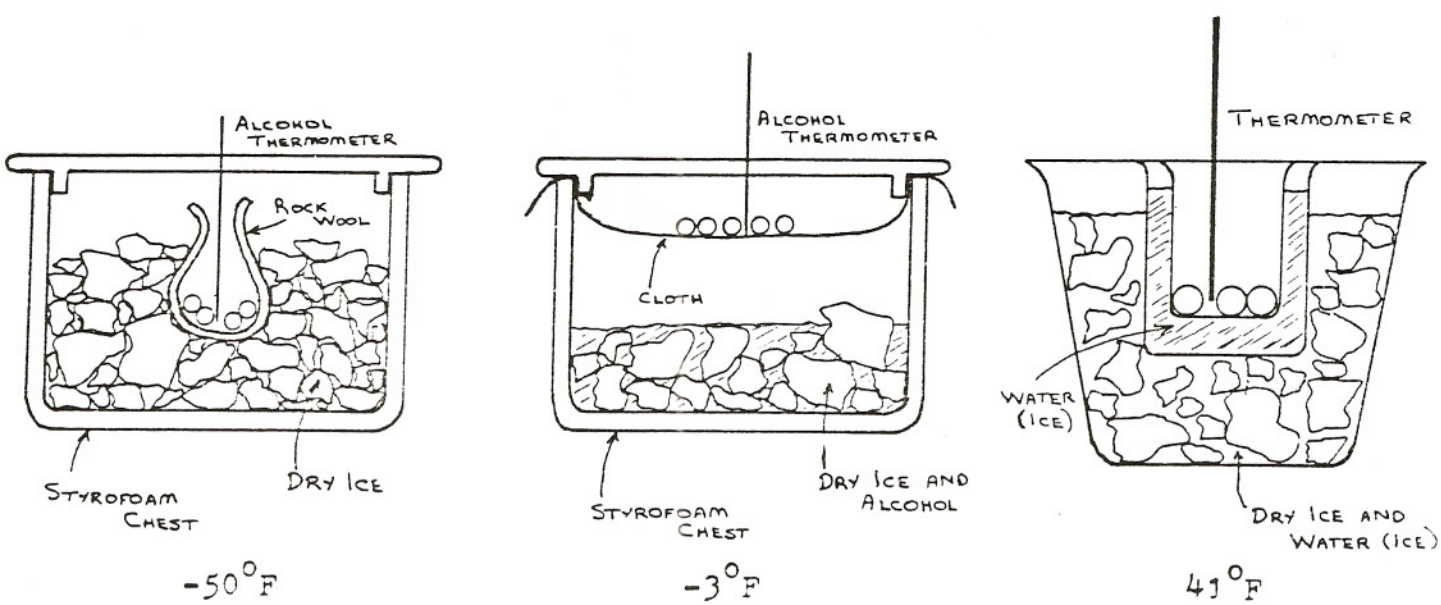


FIGURE 3. Low-temperature controlled environment systems for engines.

DATA

The data from this project are presented in two forms; Tables 1 and 2 summarize all test results, while the original data, in the form of chart recorder output, are presented for several representative test firings. Several comments concerning the interpretation of the recorder output are in order; these are given below, in a column-by-column explanation of the headings in the Tables.

In Table 1, "peak pressure" is the maximum chamber pressure developed at any point during the particular test.

"Average pressure" is more complex. It may be noted that the pressure (and thrust) output curves are characterized by a fairly slow, exponential-type buildup, followed by a gradual rising, linear increase, then a very sharp decrease. Average pressure is the average of the pressures at the beginning and end of the linear portion of the output. The linear portion usually represents more than 80% of the burn time, so it is felt that this average value closely represents the value which determined burning rate.

"Average thrust" is the average of the thrusts at the beginning and end of the period over which the above-mentioned average pressure was taken.

"Total impulse" was obtained by integrating the area under the thrust-time curve, from the point where the thrust rises to about 4% of its average value to the point where it falls back to 4%.

"Propellant weight" was the difference between the pre-fire and post-fire weights of the propellant charges. It includes an error of a few percent due to the partial (but very limited) consumption of the cardboard casing during the burn.

"Burning rate" is another complicated measurement; the pressure-time curves do not rise from or fall to zero vertically, but rather slope upward. At what point can one say with some confidence that the entire surface of the propellant grain has been ignited and is burning smoothly? For the purposes of rate measurement, the burn time was assumed to begin when the chamber pressure reached 50% of that at the low end of the linear portion of the pressure curve, and was assumed to end when the pressure fell to 50% of that at the high end. Approximations of this sort are standard in professional rocketry. The propellant grains were 2.45"$\pm$.05" in length.

"Specific impulse" was obtained by dividing the delivered total impulse by the measured propellant weight burnt off.

"Theoretical optimum thrust coefficient (C_F)" is the one that would be produced by an ideal nozzle operating at the measured chamber pressure with an ideal exhaust gas, and with the actual nozzle area ratio used in the test.

"Delivered thrust coefficient" was obtained by rearranging the expression: $F = C_F p_c A_t$ into the form: $C_F = F/p_{avg} A_t$, and substituting in the measured average thrust and pressure, and the nozzle throat area. If the exhaust gases are behaving in an ideal manner, it should equal the theoretical C_F.

"Nozzle throat diameter" is the source of the error in this project; black powder combustion products condensed on the nozzle throat during each firing, reducing the diameter somewhat from the value originally measured (the value marked with an asterisk). The test results in Table 1 are grouped according to the original throat diameter of the nozzle insert used; four tests were conducted with each insert. An effort was made after each test to remove the

deposits from the previous one, with unknown success. It was not possible to measure the throat diameter accurately after each test, so this value had to be approximated after the fact by a technique discussed in the "Analysis" section of this report.

"Nozzle area ratio" is the ratio of the area of the nozzle exit to that of the nozzle throat. The exit diameters did not change during the tests; variations in the area ratio are based on the estimated changes in throat area.

"Burn area to throat area ratio" is exactly what it says. The propellant was assumed to burn with a flat surface of constant area.

"Average pressure to atmospheric pressure ratio" was calculated for use in determining theoretical thrust coefficients.

In Table 2, "temperature" is that at which the engine was stored.

"Burning rate" was obtained by dividing the estimated grain length (1.6") by the measured burn time. The exact value of this quantity is not important, only the relative values for different temperatures. In order to eliminate the effects of slow or uneven ignition, burn time was considered to begin only when the thrust reached 0.5 lb. It ended when the grain burned through and ejected the engine from the holder.

"Peak thrust" is the maximum reached during the test.

"Sustainer thrust" is the average value of the thrust during the long plateau period following the initial peak.

"Total impulse" was obtained by integrating the area under the thrust-time curve, with appropriate corrections made for weight loss effects.

$C_{F_{Theor}}$ should be $.98 C_{F_{Func}} - \frac{.98 P_a}{P_c} \epsilon$

test #	peak pressure (psig) / psia P_c	avg pressure (psig)	avg thrust (lbs)	total impulse (N-sec)	prop weight (gms)	burn rate (in/sec) / $\frac{}{I_{sp}}$	specific imp (sec)	C_F(theoretical optimum) $C_{F_{Theor}}$	C_F(delivered)	$C_{F(del)}/C_{F(opt)}$	nozzle throat diam (in)	nozzle area ratio	burn area/ throat area	avg pressure/ atmos pressure
1	60	55	0.60	9.9	17.2	0.69	58	0.87	0.87	1.00	.112*	(3.2)	20	4.65
2	65	55	0.62	10.1	17.5	0.69	59	0.87	0.90	1.03	.112	3.2	20	4.65
3	66	56.5	0.68	10.2	16.9	0.72	62	0.86	0.98	1.14	.110	3.3	20.5	4.85
4	65	58	0.66	10.2	17.7	0.71	59	0.86	0.95	1.10	.110	3.3	20.5	4.85
5	39	34	0.48	8.5	17.1	0.60	51	0.85	0.69	0.81	.135*	(2.0)	13.7	3.26
6	40	38	0.47	8.6	17.2	0.64	51	0.89	0.68	0.76	.129	2.2	15	3.53
7	41	37	0.49	8.9	17.2	0.63	53	0.88	0.71	0.81	.130	2.1	14.6	3.46
8	40	36	0.50	8.7	17.2	0.64	52	0.87	0.72	0.83	.132	2.1	14.2	3.40
9	9	8	0.17	4.0	17.0	0.43	24	0.53	0.28	0.53	.183*	(1.5)	7.5	1.53
10	11	9	0.27	4.6	17.2	0.42	27	0.53	0.44	0.83	.180	1.5	7.7	1.60
11	10	8	0.19	4.3	17.4	0.41	25	0.53	0.32	0.60	.183	1.5	7.5	1.53
12	11	9	0.19	4.6	17.6	0.43	27	0.53	0.31	0.59	.180	1.5	7.7	1.60
13	140	125	0.70	9.9	17.4	0.80	58	1.15	0.90	0.78	.084*	(4.2)	35.5	9.35
14	150	135	0.85	11.4	17.4	0.81	67	1.17	1.10	0.94	.081	4.5	38	10.0
15	158	145	0.93	11.6	17.2	0.83	69	1.19	1.18	0.99	.079	4.7	40.2	10.6
16	155	140	0.88	11.0	17.1	0.85	66	1.18	1.13	0.95	.080	4.6	39	10.3
17	360	310	0.97	11.6	17.0	0.87	70	1.35	1.21	0.90	.056*	(6.9)	80	21.6
18	370	340	1.17	13.8	16.5	0.92	86	1.37	1.39	1.02	.055	7.1	81	23.6
19	405	365	1.20	13.2	17.8	0.97	76	1.38	1.38	1.00	.054	7.4	86.5	25.3
20	415	385	1.35	15.4	16.9	0.95	93	1.38	1.53	1.11	.053	7.6	90	26.7
21	295	285	1.02	11.8	16.6	0.91	73	1.37	1.24	0.90	.059*	(5.4)	72	20.0
22	320	285	1.03	12.1	16.8	0.89	74	1.37	1.25	0.91	.059	5.4	72	20.0
23	365	345	1.31	15.0	17.2	0.94	89	1.38	1.53	1.11	.055	6.3	88	24.0
24	355	320	1.24	15.0	17.0	0.91	90	1.37	1.44	1.05	.057	6.0	77	22.3

TABLE 1. Summary of test data. Pressures above are gauge; add 14.7 psi to obtain absolute.

$\gamma = 1.3$

11-15-77

C_{Func}

1.5477

1.4609

1.3929

1.5886

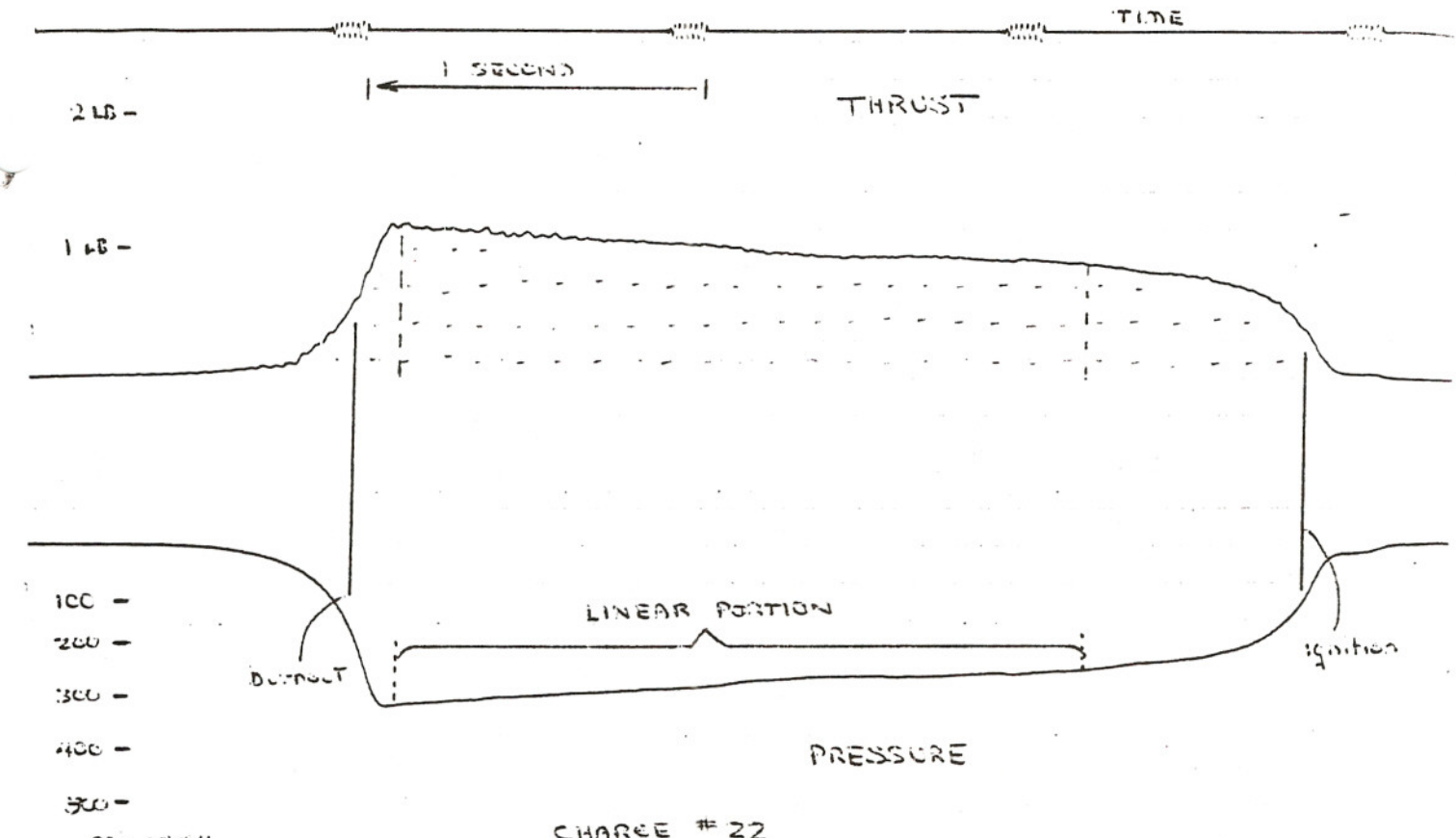
1.6508

1.6218

Doubts here due to propellant blow thru. Engines should have had epoxy clay plugs - all

temperature (°F)	burn time (sec)		max thrust (lb)		sust thrust (lb)		total impulse (N-sec)	
-50	2.37		2.65		0.75		9.08	$\bar{t}_b = 2.36$ 0.03
	2.34		2.40		0.75		9.12	$\bar{F}_p = 2.47$ 0.16
	2.41		2.50		0.75		9.11	$\bar{F}_{sus} = 0.75$ 0.01
	2.34		2.47		0.77		9.25	
	2.34	2.36	2.55	2.47	0.74	0.75	8.87	$\bar{I}_T = 9.00$ 0.18
	2.34		2.71		0.72		8.68	
	2.34	.03	2.26	0.16	0.74	0.01	8.83	
	2.39		2.24		0.75		8.99	
	2.37		2.45		0.75		9.11	
-3	2.17		2.70		0.82		8.79	
	2.19		2.46		0.82		8.94	
	2.26		2.50		0.81		9.06	
	2.17	2.20	2.76		0.81		8.96	
	2.22		2.35		0.82		8.92	
	2.17	0.03	2.55		0.82		8.92	
	2.19		2.36		0.84		8.89	
	2.22		2.73		0.82		9.07	
41	2.13		2.69		0.85		9.00	
	2.11		2.75		0.85		9.03	
	2.11		2.78		0.89		9.28	
	2.13		2.82		0.84		9.05	
	2.17		2.47		0.83		9.05	
	2.15		2.94		0.86		9.34	
	2.11		2.72		0.89		9.13	
	2.13		2.82		0.85		9.31	
	2.15		2.82		0.88		9.37	
78	2.06		2:80		0.88		9.09	
	2.06		3.00		0.88		9.00	
	2.02		3.15		0.91		9.17	
	2.02		3.08		0.88		9.17	
	2.06		2.68		0.88		9.19	
	2.04		3.00		0.90		9.14	
	2.06		2.94		0.84		8.78	
	2.06		2.60		0.85		8.70	
122	1.98		3.05		0.96		9.66	
	1.98		2.97		0.96		9.57	
	1.94		3.27		1.00		9.68	
	1.96		3.02		0.96		9.53	
	1.96		3.13		0.95		9.63	
	1.98		3.00		0.96		9.56	
	1.96		2.96		0.95		9.54	
	1.98		2.95		0.96		9.62	
	1.96		2.84		0.96		9.52	
158	1.84		3.15		1.00		9.44	
	1.89		3.02		0.98		9.60	
	1.91		3.15		0.98		9.40	
	1.91		3.20		1.00		9.42	
	1.84		3.31		1.03		9.53	
	1.89		3.46		0.99		9.35	
	1.91		3.28		1.00		9.45	
	1.91		3.01		0.98		9.29	
	1.91		3.29		1.00		9.20	

TABLE 2. Results of temperature-variation tests.



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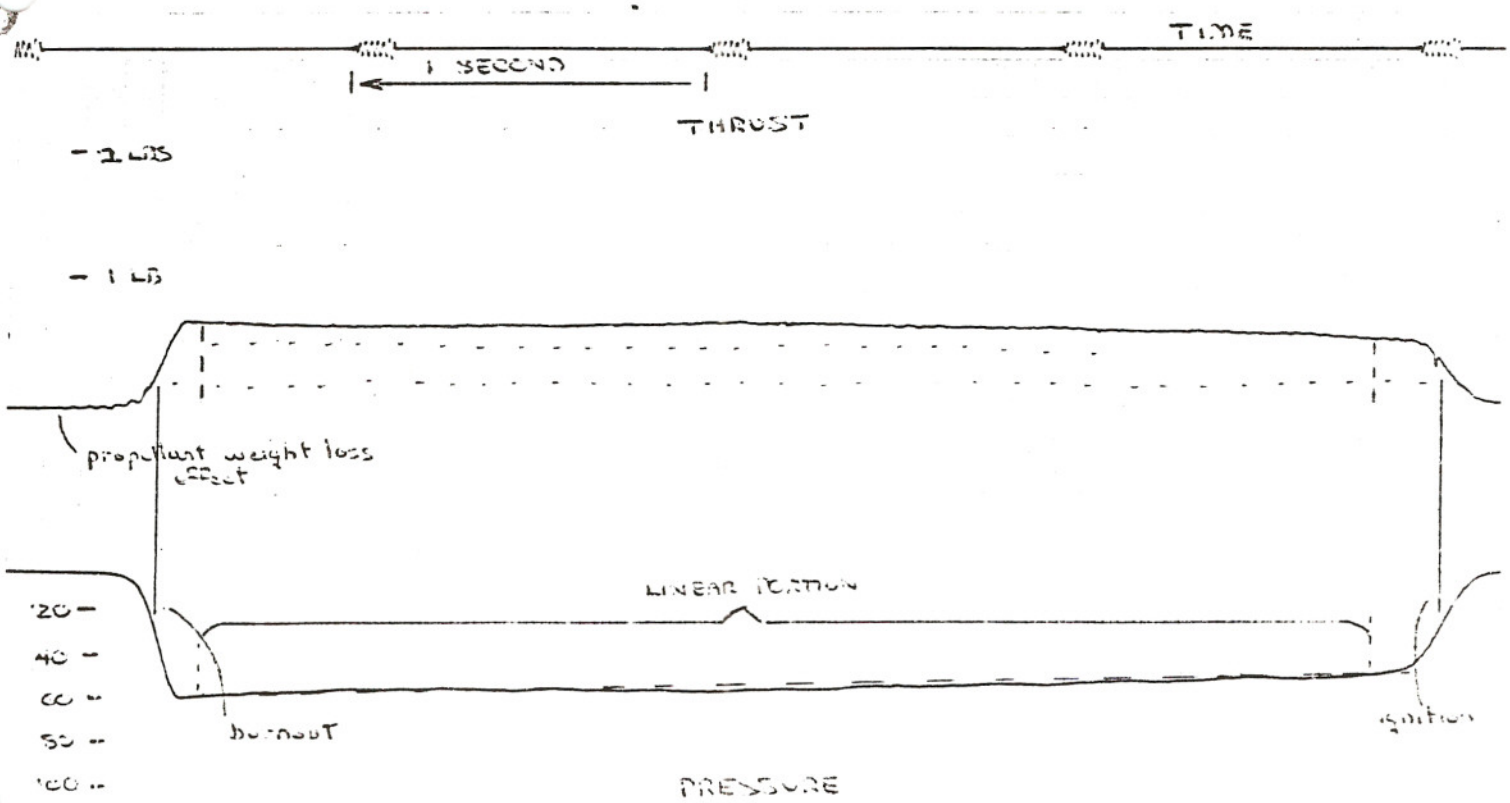

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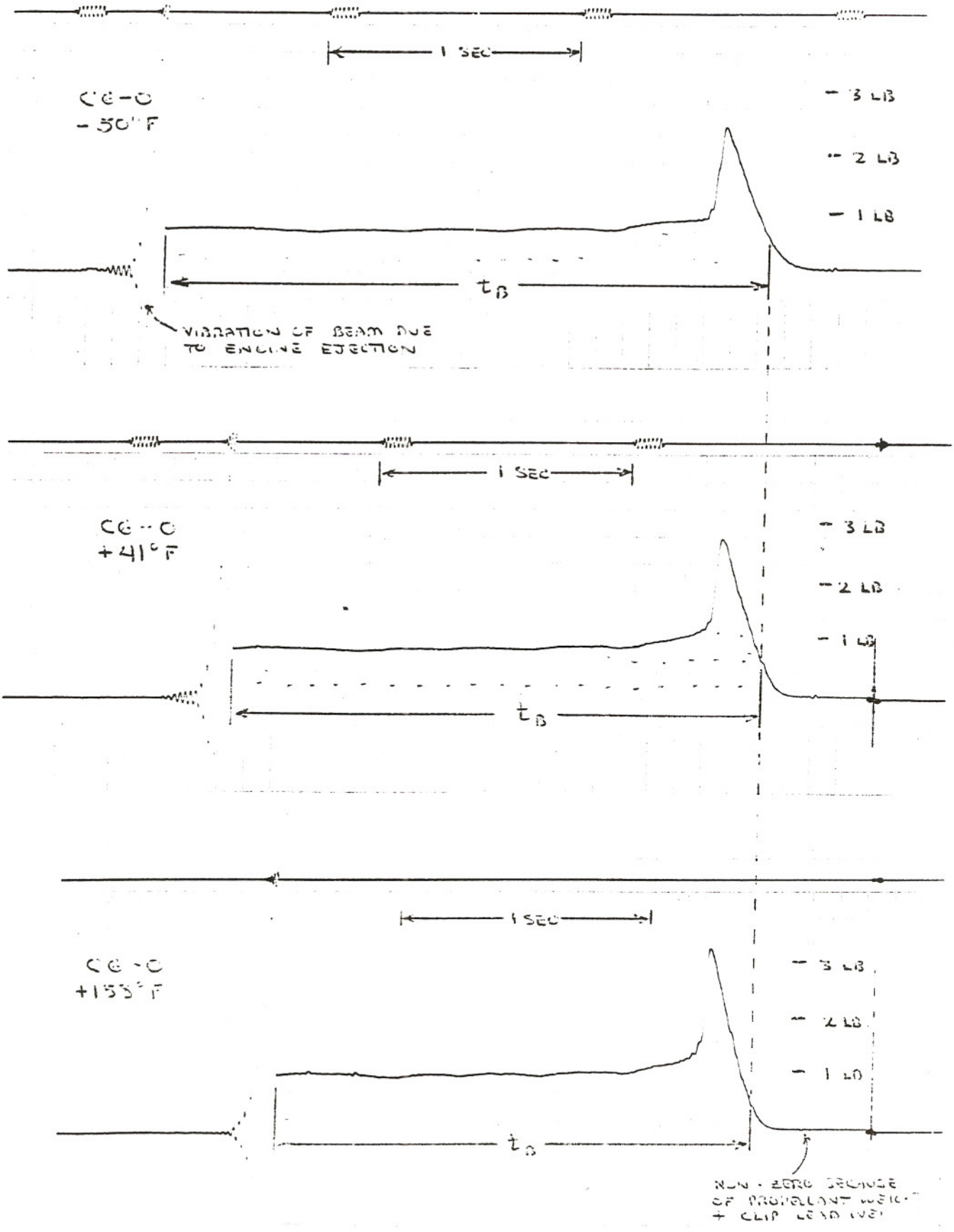

1 SEC
CG-C
-50° F
3 LB
2 LB
1 LB
t_B
VIBRATION OF BEAM DUE TO ENGINE EJECTION
1 SEC
CG-C
+41° F
3 LB
2 LB
1 LB
t_B
1 SEC
CG-C
+153° F
3 LB
2 LB
1 LB
t_B
NON-ZERO BECAUSE OF PROPELLANT WEIGHT + CLIP LEAD WEI

ANALYSIS OF DATA

The results of the experimental work in this project are clearly displayed in Graphs 1-4.

Graph 1 shows the variation of the linear burning rate of Estes black powder with the absolute chamber pressure at which it is burned. This relationship is generally expressed by Vielle's Law:

$$r = ap_c^n.$$

Here, "a" is a function of both the properties of the propellant and the pre-firing grain temperature, while "n" is strictly determined by the composition of the propellant. The form of this law leads to a log-log presentation, where $\log r = \log a + n \log p_c$, and

$$n = \frac{\log r - \log a}{\log p_c}$$

Here, log a is a constant if the ambient temperature is constant (as it was in the pressure-variation tests), so the burning rate exponenet "n" is given by the slope of a log-log plot of r versus p_c.

The data from this project indicate clearly that black powder undergoes a change in the value of its rate exponent at approximately 75 psia. Below this pressure, a curve-fit of the data by the method of extended differences (see Appendix I) gives a value of

$$n = 0.462 \pm .031 \qquad a = .099$$

with the line passing through the point r=0.58 in/sec, p_c=49 psia. This point was calculated statistically; the value of "a" was not taken directly from the data, but was calculated so that the line with the above slope "n" would pass through this point. Above 75 psia, a similar calculation gives

$$n = 0.133 \pm .022 \qquad a = .408$$

with this line passing through r=0.87 in/sec, p_c=299 psia. A change of this sort in "n" is typical of almost all solid propellants; it

simply indicates some sort of transition between dominant mechanisms in the combustion process. Nothing is known about the combustion of black powder at typical rocket chamber pressures, so speculation about the reasons for this shift would be pointless.

Graph 2 shows the nozzle size required to obtain a particular chamber pressure (and thus burning rate, thrust, etc.) from a given grain. The graph was drawn using only 6 data points, those from the first test-firing with each of the nozzle inserts, because of the previously-mentioned problem with residue buildup on the nozzle throats. It was felt that only on these tests could the nozzle throat size be stated to adequate precision for a graph of this type. The six points are almost exactly collinear; the exact curve-fitting calculations are carried out in Appendix I.

A few words should be said at this point concerning the techniques whereby those nozzle throat diameters in Table 1 not marked by an asterisk were estimated. The method was simple; Graph 2 was first drawn up on the basis of the presumably accurate area ratios from the first firing with each nozzle, then was entered with the average pressures from subsequent firings. The area ratios corresponding to these pressures were obtained and worked backwards to give an estimated nozzle throat diameter.

Graph 3 deals with the pressure dependence of the efficiency of the black powder combustion process. Experimental scatter here precludes making very definite statements, but it is clear at least that, as the chamber pressure increases, so does the efficiency of this propellant as measured by its specific impulse. This suggests that its frequently-observed tendency to eject substantial quantities of unburned material is reduced by an increase in pressure. This

could arise from two mechanisms: either the burning rate becomes rapid enough that the particles are consumed before leaving the nozzle (at least in the very long motor used in this project), or there is a gradual transition in the combustion process so that the number and/or size of these particles is decreased, indicating a shift in the location of the exhaust equilibrium. In either case, the flow becomes more nearly one-phase and ideal, causing the delivered C_F to approach the theoretically (ideally) possible value. This, of course, increases thrust and I_{sp}, as less propellant energy is wasted by being released in the exhaust. The precise reason for the sudden increase in I_{sp} at pressures over 300 psia is not clear; there is no measured transition of burning rate exponent there to indicate a simultaneous change in the combustion process.

The temperature-variation tests in this project were analyzed statistically to obtain two important quantities: the variation of thrust, and the variation of burning rate with initial grain temperature. The mean values of the parameters in Table 2 are presented below:

From Nelson 2/4/75 Thrust .134 ± .009 %/°F Gauge pressure .100 ± .006 %/°F

Temp (°F)	Burn Rate (in/sec)	Max Thrust (lb)	Sust Thrust (lb)	Tot Imp (N-sec)	# of tests
-50	.678	2.47	0.75	9.00	9
-3	.727	2.55	0.82	8.94	8
41	.751	2.76	0.86	9.17	9
78	.780	2.91	0.88	9.03	8
122	.816	3.02	0.96	9.59	9
158	.847	3.21	1.00	9.41	9

The variation of burning rate with this temperature (for constant burning-to-throat area ratio) is conventionally expressed by:

$$(\pi_r)_K = \frac{1}{n} \sum^{n} \frac{\log\left(r_{n+1}/r_n\right)}{T_{n+1} - T_n}$$

This can be transformed into a modification of Vielle's Law:

$$r = a e^{(\pi_r)_K (T - T_{ref})} p_c^n$$

The values of burning rate versus temperature are plotted in Graph 4, with the methods of Appendix 1 being used to determine that

$$(\pi_r)_K = .00103 \pm .00003$$

The values of "a" and "n" from the pressure-variation measurements were obtained at approximately 78°F (T_{ref}), so the complete expression for rate is:

$$r = .099\ e^{.00103(T-78)}\ p_c^{.462} \qquad (p_c \leq 75 \text{ psia})$$

$$r = .408\ e^{.00103(T-78)}\ p_c^{.133} \qquad (p_c > 75 \text{ psia})$$

The temperature variation of thrust is conventionally expressed as a percentage per degree

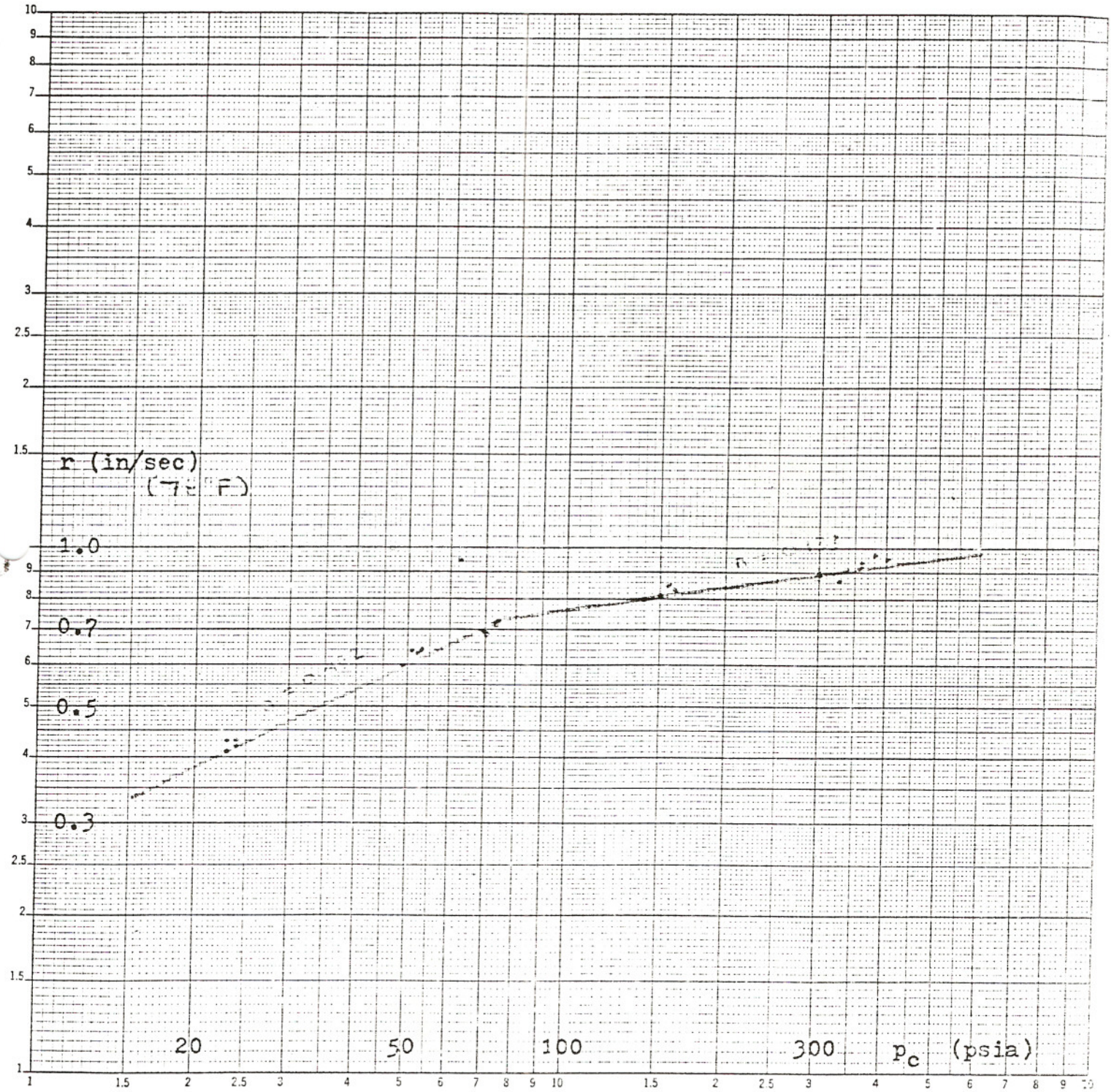
$$\tau = \frac{1}{n} \sum^{n} \frac{F_{n+1} - F_n}{F_n (T_{n+1} - T_n)}$$

The measured variations of both peak and sustainer thrusts were combined for statistical analysis to give

$$\tau = 0.136\ (\pm .020)\ \%/^{\circ}F$$

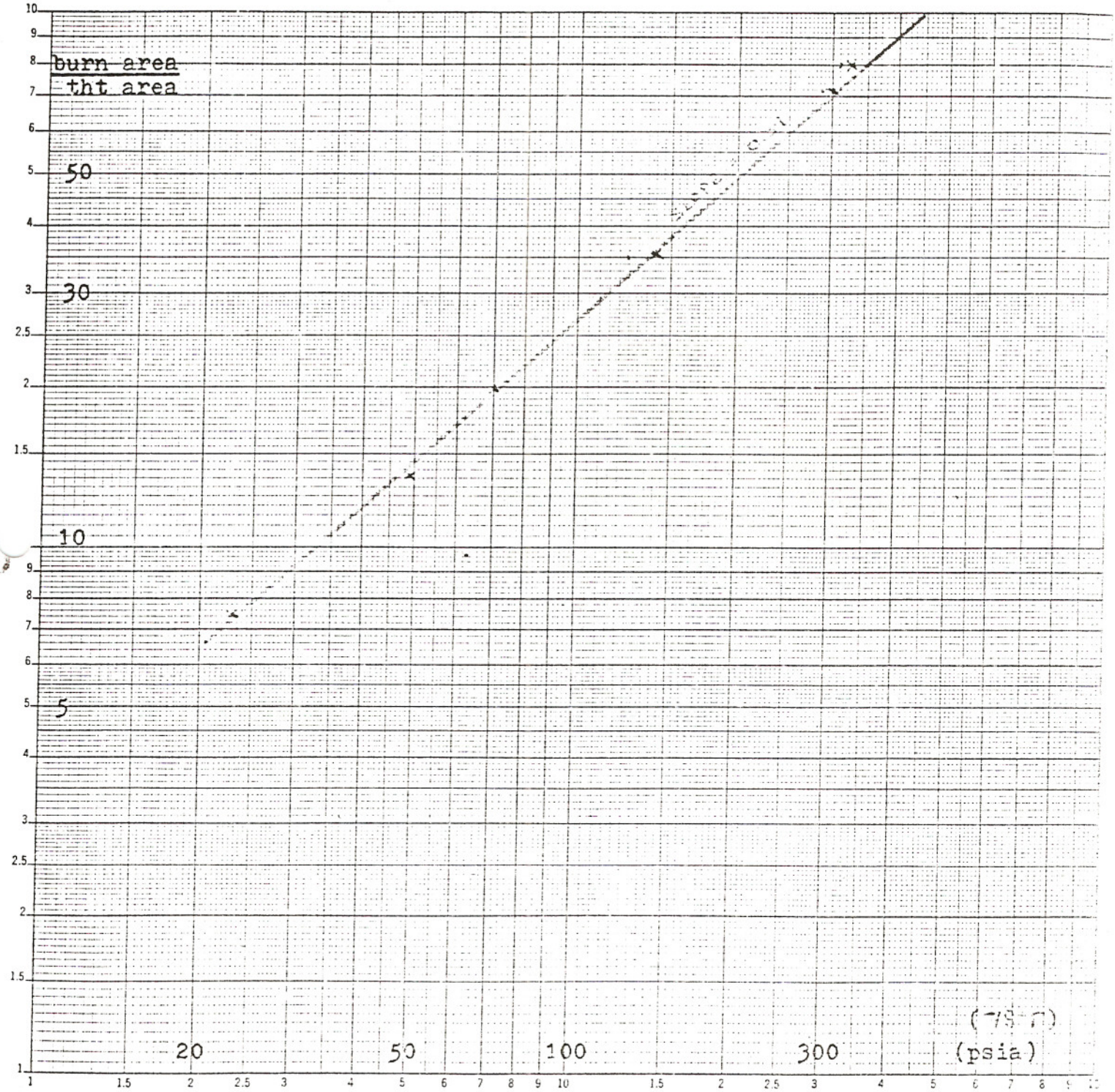
This coefficient should be applied to manufacturers' data to obtain the thrust under various temperature conditions, assuming that these data were obtained at 70°F.

The variation of total impulse with temperature was not smooth enough to analyze meaningfully; it does, however, seem to increase substantially (about 5%) at very high temperatures (greater than 110°F), but remain constant for all lower temperatures.

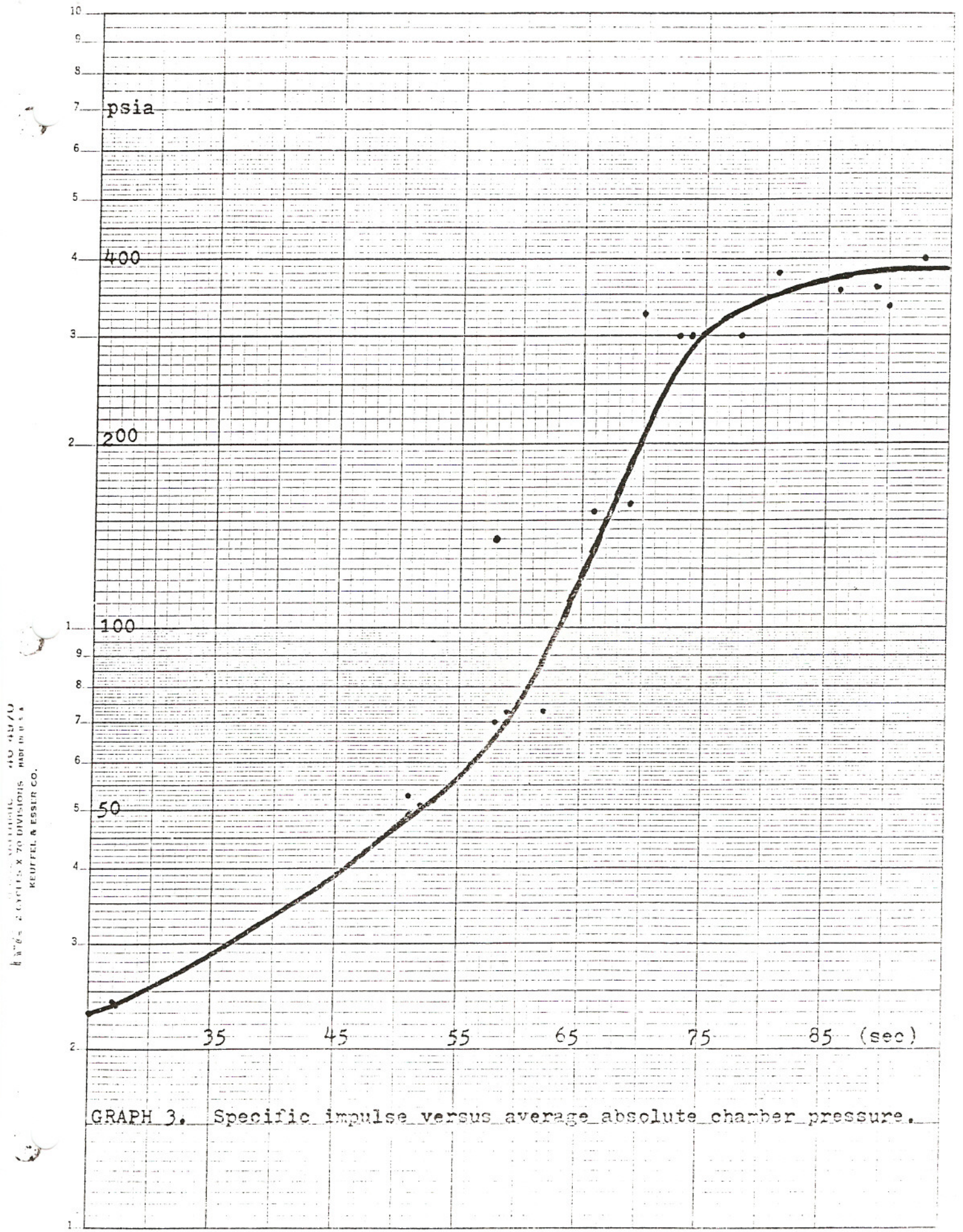


GRAPH 1. Burning rate versus average absolute chamber pressure.

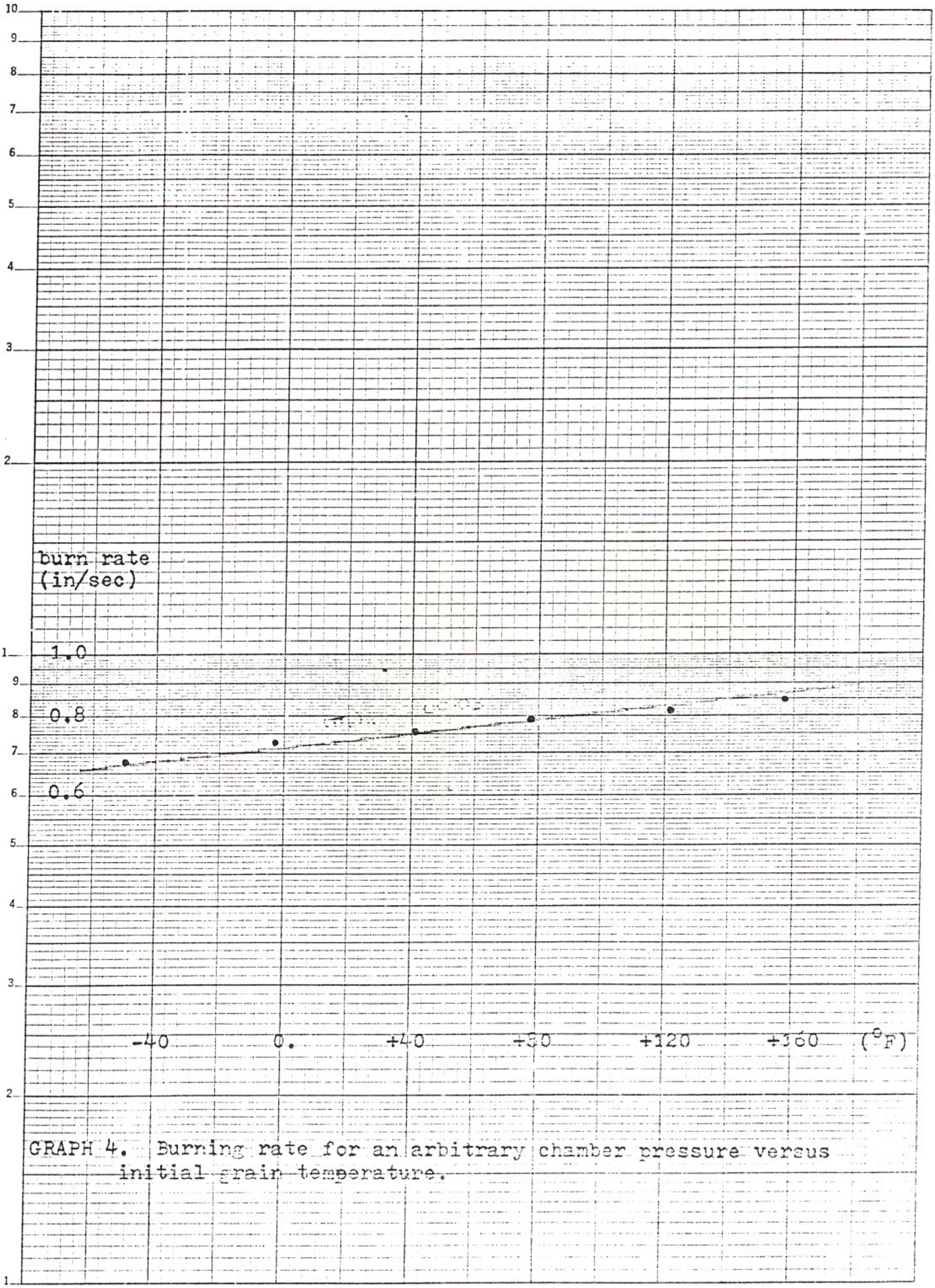
Dividing my D's by 1.3 (used here to correct for concave burning surface) to correspond with Tsiu's my points agree nicely with his Kn curves. MB



GRAPH 2. Absolute average chamber pressure vs. burn-to-throat area ratio.



GRAPH 3. Specific impulse versus average absolute chamber pressure.



GRAPH 4. Burning rate for an arbitrary chamber pressure versus initial grain temperature.

CONCLUSIONS

This project has met its goal of precisely measuring the mathematical interrelationship among chamber pressure, burning rate, and burn-to-throat area ratio for Estes black powder rocket propellant, and also the variation of its burning rate and thrust with ambient temperature. This information is essential for the rapid and precise development of new engine designs, and is of use to anyone interested in the internal ballistics of these engines. The measurement of the temperature variation effects makes possible more exact altitude prediction for very hot or very cold weather conditions. The limited quantity of available data prevented the development of exact relationships among chamber pressure, grain temperature, and specific impulse, but the I_{sp} was definitely shown at least to increase in some manner with increasing pressure and temperature.

REFERENCES

Barrere, Marcel et al, Rocket Propulsion, New York, Elsevier Publishing, 1960.

Cook, Nathan, and E. Rabinowicz, Physical Measurement and Analysis, Reading, Addison-Wesley Publishing, 1963.

Sutton, George, Rocket Propulsion Elements, New York, John Wiley, 1963.

APPENDIX I

Curve Fitting by the Method of Extended Differences.

The method of extended differences is a simple means of obtaining the "best" straight line through a set of presumably linear data points. It gives good results whether the experimental error lies in the x or the y coordinates, or both.

As a simple example of the method, the six points in Graph 2 are selected and arranged in order of ascending value of one of the co-ordinates (in this case, the area ratio).

Kn (area ratio)	p_c(avg)	log Kn	log p_c	n
7.5	23	0.875	1.361	1
13.7	49	1.136	1.690	2
20.0	70	1.303	1.845	3
35.5	140	1.550	2.146	4
72.0	300	1.857	2.477	5
80.0	325	1.903	2.512	6

The desired plot is believed to be linear on a log-log plot, so it was necessary to take the logarithms of the data point coordinates. These points are now divided into two equal groups, n=1-3 and n=4-6, and the differences in coordinates between corresponding points in the two groups are obtained, then the slopes of the lines connecting these pairs of points.

$\Delta y = (Kn_{n+3} - Kn_n)$	$\Delta x = (p_{c_{n+3}} - p_{c_n})$	$\Delta y/\Delta x$	n
0.675	0.785	0.86	1
0.721	0.787	0.92	2
0.600	0.667	0.90	3
	arithmetic mean =	0.89	

The arithmetic mean (average) of these slopes is the slope of the "best" line through the points. The difference between the mean and each of the component slopes is then taken, and the absolute values of these differences summed. This comes out to 0.07. To obtain the standard error of the mean slope, one substitutes the mean difference (0.07 divided by 3, or 0.023) into the formula:

$$E = 1.25\ MD/\sqrt{N-1}$$

Here, E is the standard error, MD is the mean difference (.023), and N is the number of $\Delta y/\Delta x$ values averaged to obtain the mean slope; in this case, 3. One thus gets a standard error of 0.02. The mean slope is $0.89 \pm .02$.

This calculation has determined only the slope of the "best" line; to obtain a point through which it should pass, one takes the centroid of all the data points, or

$$Kn = \sum^{n} Kn\ /n \qquad\qquad p_c = \sum^{n} p_c\ /n$$

This gives the point Kn = 38.1, p_c = 151 psia.

This same method was used to obtain the burning-rate-versus-pressure lines, the only difference being in the vastly greater number of data points.